quantum technologies V

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the course: structure

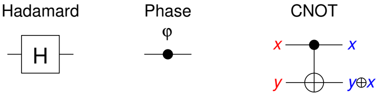
- I. Introduction to QTech (1h)
- II. Math preliminaries (2h)
  linear algebra, vector spaces, Dirac notation, operators etc
- III. Quantum mechanics: postulates (2h)
- IV. Quantum information: basic concepts (3h)
  qubit, Bloch sphere, Pauli matrices, quantum gates, no-cloning, density matrix
- V. Entanglement (2h)
  Bell, GHZ states, quantum protocols (teleportation, entanglement swapping)
- VI. Implementations (2h)
  photonic qubits, interference, photon sources, linear-optics gates, detection
- VII. Aplications: quantum cryptography and quantum communications (2h)
  QKD protocols, quantum networks, examples

recap

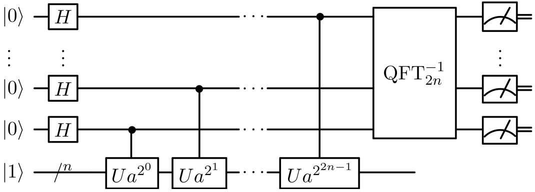
- no-cloning:
  an unknown q. state cannot be cloned

$$|\psi\rangle|0\rangle \overset{U}{\nrightarrow} |\psi\rangle|\psi\rangle$$

- universality: $\{H, P_\varphi, CNOT\}$



- 1- and 2-qubit gates: building blocks to perform any q. algorithm



I. entanglement

plan

- entanglement: definition, examples
- EPR-Bell states, GHZ, W
- quantum protocols: teleportation, entanglement swapping
- Bell-CHSH inequality

what is entanglement?

entangled = not separable

$$|\psi\rangle_{AB} \neq |\phi_1\rangle_A \otimes |\phi_2\rangle_B$$

- cannot describe it as states of separate particles
- quantum correlations - stronger than classical

the whole is more than the sum of its parts

Schrödinger

on entanglement

entanglement is not one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought

By the interaction the two representatives [the quantum states] have become entangled. Another way of expressing the peculiar situation is:

the best possible knowledge of a whole does not necessarily include the best possible knowledge of all its parts

E. Schrödinger, *Discussion of Probability Relations Between Separated Systems*, Proc. Camb. Philos. Soc. **31**, 555 (1935); **32**, 446 (1936)

EPR-Bell states

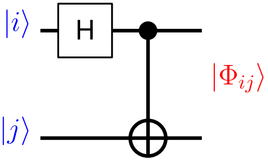
Bell basis for two qubits

$$|\Phi^+\rangle \equiv |\Phi_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\Phi^-\rangle \equiv |\Phi_{10}\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|\Psi^+\rangle \equiv |\Phi_{01}\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|\Psi^-\rangle \equiv |\Phi_{11}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$



- orthogonality:

$$\langle\Phi_{ij}|\Phi_{kl}\rangle = \delta_{ik}\delta_{jl}$$

- maps the computational basis to the Bell basis: $|i\rangle|j\rangle \mapsto |\Phi_{ij}\rangle$

local equivalence

all Bell states are locally equivalent

Exercise. Show that all Bell states are locally equivalent.

[starting from $|\Phi_{00}\rangle$ we can obtain all other $|\Phi_{ij}\rangle$ by applying only local unitaries.]

$$|\Phi_{ij}\rangle = U_i \otimes U_j \, |\Phi_{00}\rangle$$

$$\begin{array}{ccc} |\Phi_{00}\rangle & \xrightarrow{U_1} & |\Phi_{10}\rangle \\ U_2 \downarrow & & \downarrow U_3 \\ |\Phi_{01}\rangle & \xrightarrow{U_4} & |\Phi_{11}\rangle \end{array}$$

find $U_1, \dots, U_4$

entanglement

two questions

- given $|\psi\rangle$, can we decide if it's entangled or not?

  if yes, how much entanglement does it have?

- why is entanglement useful?
  - quantum computation/algorithms: Shor, Grover etc
  - quantum protocols: teleportation, entanglement swapping
  - quantum repeaters

entanglement

separability criterion: 2 qubits

$$|\psi\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$$

Theorem

$$|\psi\rangle \textit{ is separable } \Leftrightarrow C = 0$$

concurrence

$$C = 2\,|a_{00}a_{11} - a_{01}a_{10}|$$

Proof

$$\begin{aligned} |\phi_1\rangle \otimes |\phi_2\rangle &= (a|0\rangle + b|1\rangle) \otimes (c|0\rangle + d|1\rangle) \\ &= ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle \end{aligned}$$

etc

entanglement

properties

- $0 \leqslant C \leqslant 1$
- $C = 1$ maximally entangled states
- entanglement is invariant under local unitaries

$$C(U_1 \otimes U_2 \, |\psi\rangle) = C(|\psi\rangle)$$

Corollary: cannot create entanglement by acting locally on a separable state

entanglement requires an interaction between qubits

concurrence

examples

compute C for the states

1. $|\psi_1\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$ $\quad C =$
2. $|\psi_2\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle - |11\rangle)$ $\quad C =$
3. $|\psi_3\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |01\rangle)$ $\quad C =$
4. $|\psi_4\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ $\quad C =$
5. $|\psi_5\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ $\quad C =$
6. $|\psi_6\rangle = \cos\alpha|00\rangle + \sin\alpha|11\rangle$ $\quad C =$

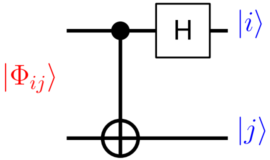
Bell state measurement (BSM)

$$|00\rangle = \frac{1}{\sqrt{2}}(|\Phi^+\rangle + |\Phi^-\rangle)$$

$$|01\rangle = \frac{1}{\sqrt{2}}(|\Psi^+\rangle + |\Psi^-\rangle)$$

$$|10\rangle = \frac{1}{\sqrt{2}}(|\Psi^+\rangle - |\Psi^-\rangle)$$

$$|11\rangle = \frac{1}{\sqrt{2}}(|\Phi^+\rangle - |\Phi^-\rangle)$$

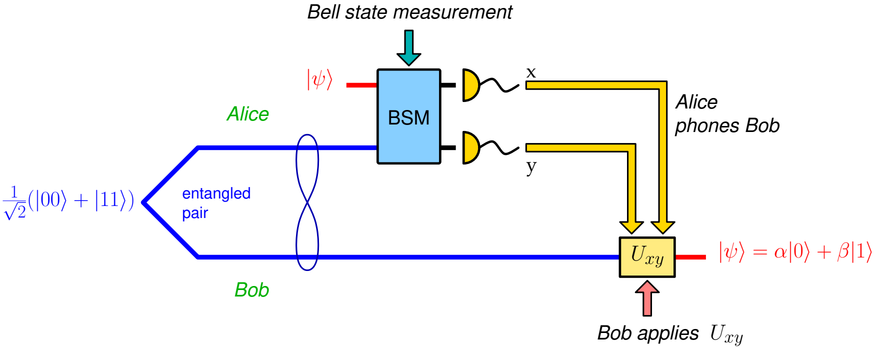


- maps the Bell basis to the computational basis: $|\Phi_{ij}\rangle \mapsto |i\rangle|j\rangle$
- crucial for teleportation

II. quantum protocols

teleportation

send an unknown state $|\psi\rangle$ over a quantum channel

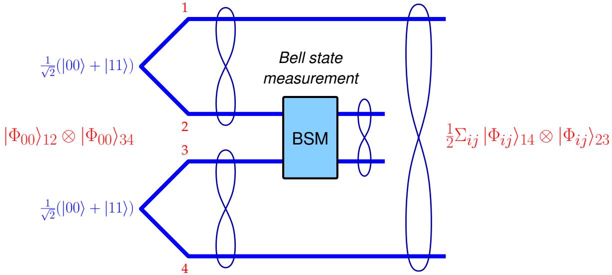


$$|\psi\rangle_1 \otimes |\Phi_{00}\rangle_{23} = \frac{1}{2}\{|\Phi_{00}\rangle_{12} \otimes |\psi\rangle_3 + |\Phi_{10}\rangle_{12} \otimes Z|\psi\rangle_3 + |\Phi_{01}\rangle_{12} \otimes X|\psi\rangle_3 + |\Phi_{11}\rangle_{12} \otimes XZ|\psi\rangle_3\}$$

entanglement swapping

teleportation of entanglement

entangle 2 systems which never met using a quantum channel

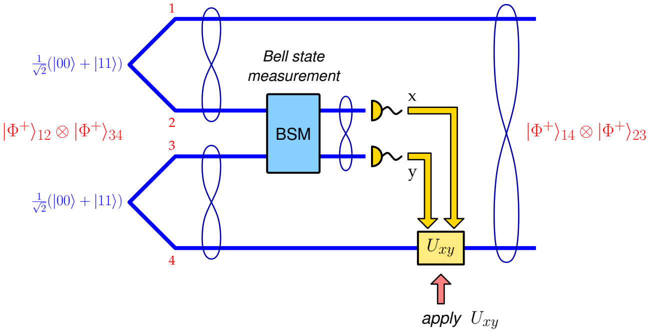


$$|\Phi_{00}\rangle_{12} \otimes |\Phi_{00}\rangle_{34} = \frac{1}{2}\sum_{ij} |\Phi_{ij}\rangle_{14} \otimes |\Phi_{ij}\rangle_{23}$$

entanglement swapping

teleportation of entanglement

entangle 2 systems which never met using a quantum channel



$$|\Phi_{00}\rangle_{12} \otimes |\Phi_{00}\rangle_{34} = \frac{1}{2} \sum_{ij} |\Phi_{ij}\rangle_{14} \otimes |\Phi_{ij}\rangle_{23}$$

entanglement: generalization

can we generalize the Bell states?

yes

1. more qubits

2. more dimensions (qudits)

entanglement: more qubits

3 qubits

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

$$|W\rangle = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)$$

n qubits

$$|GHZ_n\rangle = \frac{1}{\sqrt{2}}(|00\ldots0\rangle + |11\ldots1\rangle)$$

$$|W_n\rangle = \frac{1}{\sqrt{n}}(|10\ldots0\rangle + |01\ldots0\rangle + \ldots + |00\ldots1\rangle)$$

GHZ $\neq$ W

$$|GHZ\rangle \neq_{LOCC} U_1 \otimes U_2 \otimes U_3 \, |W\rangle$$

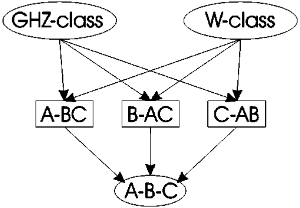


FIG. 1. Different local classes of tripartite pure states. The direction of the arrows indicates which noninvertible transformations between classes are possible.

Dür, Vidal, Cirac, *Three qubits can be entangled in two inequivalent ways*, Phys. Rev. A **62**, 062314 (2000); quant-ph/0005115

entanglement: more dimensions

two qudits

$$|\Phi_d\rangle = \frac{1}{\sqrt{d}}(|00\rangle + |11\rangle + \cdots + |d-1\ d-1\rangle) = \frac{1}{\sqrt{d}}\sum_{i=0}^{d-1}|i\rangle|i\rangle$$

there are d^2 maximally-entangled states for two qudits

hint: apply $Z_d^i X_d^j|\Phi_d\rangle$, $i, j = 0 \ldots d-1$

Z_d, X_d generalized Pauli matrices for qudits

$Z_d|i\rangle = \omega^i|i\rangle$, $\omega^n = 1$

$X_d|i\rangle = |i \oplus 1\rangle$

Bell-CHSH

... a (little) bit of history

- 1920's: Einstein vs. Bohr (the real battle of the titans)
- 1935: EPR (Einstein, Podolsky, Rosen)

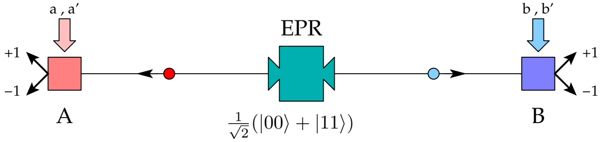
  ⋮ [dark ages, shut-up-and-calculate]
- 1964: Bell: testing philosophical assumptions in the lab !
- 1990's: turning paradoxes into technologies

  [from philosophical debates to technological updates]

Bell-CHSH violated $\Rightarrow$ *secure quantum communications*

Bell-CHSH

... or why quantum $\neq$ classical



classical (local realism)

$|S| \leqslant 2$

quantum (Bell states)

$|S| = 2\sqrt{2} \approx 2.828$

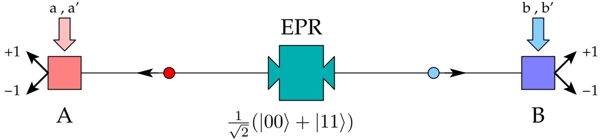
$\Downarrow$

QM violates local realism

$\Downarrow$

quantum correlations: stronger than classical

Bell-CHSH



$$S = E(a,b) + E(a',b) + E(a',b') - E(a,b')$$

$$E(a,b) = \frac{N_{++} - N_{+-} - N_{-+} + N_{--}}{N_{tot}}$$

$$(a,b,a',b') = (0, 22.5^\circ, 45^\circ, 67.5^\circ)$$

- classical: $|S| \leqslant 2$
- quantum: $|S| = 2\sqrt{2} \approx 2.828$

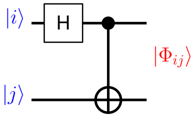
you'll do the experiment in the lab!

summary

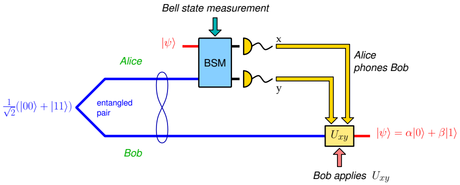
- "entanglement [...] *the characteristic trait of quantum mechanics*" (Schrödinger)
- both a mystery and a resource (... and much more)
- Bell states

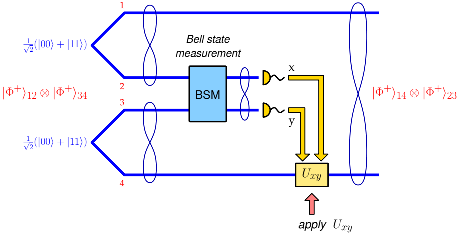
$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$

$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$



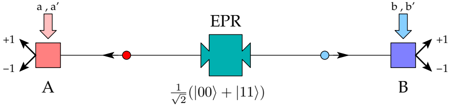
- teleportation, entaglement swapping





summary

- Bell-CHSH



- classical: $|S| \leqslant 2$
- quantum: $|S| = 2\sqrt{2} \approx 2.828$
- secures quantum communications

Thank you!

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